ANSWER OF MODEL PAPER FOR IIT-JEE

PHYSICS

1. **Ans.** both (i) and (ii) are correct. **Reason:** The displacement of the particle is determined by the area bounded by the curve. This area is $s = \frac{\pi}{4} v_m t_0$

The average velocity is $<\vec{v}> = \frac{s}{t_0} = \frac{\pi}{4} v_m$

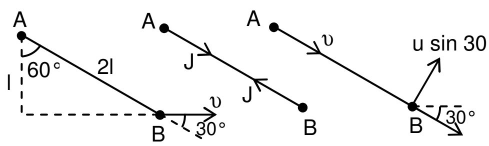
Such motion can not be realized in practical terms since at the initial and final moments the acceleration, which is slope of v-t graph is infinitely large. Hence both (i) and (ii) are correct.

2. **Ans.** $g\frac{\sin^2\theta}{1+\sin^2\theta}$. **Reason:** Let a be the acceleration of wedge down the plane and N the normal reaction between A and B. The acceleration of A will be a sin θ vertically downwards. Then equations of motion of B and A give $(N + mg)\sin\theta = ma$ …………. (i)
and $(mg - N) = ma\sin\theta$ ………… (ii)
Solving these two equations, we get
$a = \frac{2g\sin\theta}{1+\sin^2\theta}$ acceleration of A is $a_A = a\sin\theta$
$= 2g\frac{\sin^2\theta}{1+\sin^2\theta}$.

Displacement of A in 1 s is $S = \frac{1}{2}a_A(1)^2 = g\frac{\sin^2\theta}{1+\sin^2\theta}$.

3. **Ans.** $\frac{u\sqrt{3}}{4}$ **Reason:** When the string jerks tight both particles begin to move with velocity components v in the direction AB. Using conservation of momentum in the direction AB $mu\cos 30° = mv + mv$

or $v = \frac{u\sqrt{3}}{4}$. Hence the velocity of ball A just after the jerk is $\frac{u\sqrt{3}}{4}$.



4. **Ans.** $\sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$ **Reason:** Total energy of a planet in an elliptical orbit is : $E = -\frac{GMm}{2a}$ (m = mass of planet)
From conservation of mechanical energy
K.E + P.E = E.
or $\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$ or $v = \sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$

5. **Ans.** 6/7 **Reason:** At $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ sin kx_1 or sin kx_2 is not zero.

Therefore, neither of x_1 or x_2 is a node
$\Delta x = x_2 - x_1 = \left(\frac{3}{2}-\frac{1}{3}\right)\frac{\pi}{k} = \frac{7\pi}{6k}$.

since $\frac{2\pi}{k} > \Delta x > \frac{\pi}{k}$; $\lambda > \Delta x > \lambda/2$ $\left(k = \frac{2\pi}{\lambda}\right)$.

Therefore, $\phi_1 = \pi$ and $\phi_2 = k.\Delta x = \frac{7\pi}{6}$.

$\therefore \frac{\phi_1}{\phi_2} = \frac{6}{7}$.

6. **Ans.** $\frac{6}{5}$ **Reason:** Internal energy of n moles of an ideal gas at temperature T is given by
$U = \frac{f}{2}nRT$ (f = degrees of freedom)
$U_1 = U_2$. $\therefore f_1n_1T_1 = f_2n_2T_2$
$\therefore \frac{n_1}{n_2} = \frac{f_2T_2}{f_1T_1} = \frac{(3)(2)}{(5)(1)} = \frac{6}{5}$

7. **Ans.** V = -xy + constant. **Reason:** $dV = -\vec{E}.\vec{dr}$
$= -(y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$
$= -(ydx + xdy) = -d(xy)$
Integrating, we get V = -xy + constant.

8. **Ans.** x/8. **Reason:** $x = \frac{B}{M} = \left(\frac{\mu_0 i}{2r}\right)\left(\frac{1}{i\pi r^2}\right)$ or $x \propto \frac{1}{r^3}$.

i.e., x will become x/8 when radius and current both are doubled.

9. **Ans.** • $\upsilon = [k(M+m)]^{1/2}\frac{x}{m}$ • $V = \left(\frac{k}{M+m}\right)^{1/2} x$.

Reason : PE Stored in the spring $= \frac{1}{2}kx^2$.

$mv = (M + m)V$ or $V = \frac{mv}{(M+m)}$ …… (i)

After collision, KE of block + bullet in it = PE of the spring. Thus $\frac{1}{2}(M+m)V^2 = \frac{1}{2}kx^2$ which gives

$V = \sqrt{\frac{k}{(M+m)}}.x$ ……. (ii)

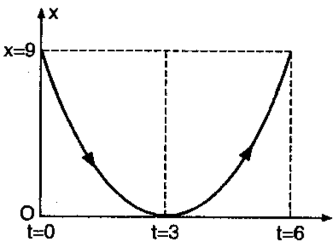
Using eq. (i) and (ii), we have $v = \frac{(M+m)V}{m}$

$= \frac{(M+m)}{m}\left(\frac{k}{M+m}\right)^{1/2} x = [k(M+m)]^{1/2} x/M$

10. **Ans.** • 1/RC • R/L • $1/\sqrt{LC}$

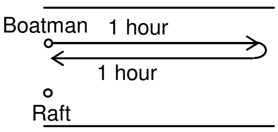
11. **Ans.** • It represents a stationary wave of frequency 60Hz. • It is the result of superposition of two waves of wavelength 3m, frequency 60Hz each travelling with a speed of 180 m/s in opposite direction.

12. **Ans.** • the intensity of emitted radiation increases • the minimum wavelength of emitted radiation decreases.
13. **Ans.** • The induced emf is zero in position II • The induced emf is anti-clockwise in position I • The induced emf is clockwise in position III. **Reason :** When the loop is completely inside the field, the ϕ = B. A = constant.
14. **Ans.** OA. **Reason:** Hooke's law is obeyed in the linear portion of the graph.
15. **Ans.** B
16. **Ans.** Copper. **Reason:** Copper is a conductor.
17. **Ans.** a current flows in the circuit for sometime and then decreases to zero.
18. **Ans.** An alternating current flows in the circuit. **Reason:** Alternating voltage implies an alternating current flows.
19. **Ans.** 4.
20. **Ans.** 6. **Reason :** According to conservation of energy,
$$\frac{hc}{\lambda_1}+\frac{hc}{\lambda_2}=RchZ^2\left(\frac{1}{1^2}-\frac{1}{n^2}\right) \text{ or } \frac{1}{\lambda_1}+\frac{1}{\lambda_2}=RZ^2\left(1-\frac{1}{n^2}\right)$$
or $\frac{1}{n^2}=1-\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}\times\frac{1}{RZ^2}$
$$=1-\frac{13330.7\times10^{-10}}{1026.7\times304\times10^{-20}\times4\times1.097\times10^7}$$
Thus $1/n^2$ = 0.0284 or n = 5.93 = 6. Hence the quantum number = 6.
21. **Ans.** 9.
22. **Ans.** 2. **Reason :** As the network AQCS is a balanced Wheatstone bridge, no current will flow through AC and hence the effective resistance of the network between QS : $R_{QS}=\frac{6\times6}{6+6}=3$ ohm and as the resistance of the loop is 1 ohm, the total resistance of the circuit, R = 3 + 1 = 4 ohm. Now if the loop moves with speed v_0, the emf induced in the loop, e = Bv_0l. So the current in the circuit $I=\frac{e}{R}=\frac{Bv_0l}{R}$. Substituting the given data, $v_0=\frac{IR}{Bl}=\frac{1\times10^{-3}\times4}{2\times0.1}$ = 2 × 10^{-2} ms^{-1} = 2 cm/s. In accordance with Lenz's law the induced current in the loop will be clockwise.
23. **Ans.** 8. **Reason :** As, r >> λ,
So $PA=\sqrt{r^2+\lambda^2+2r\lambda\cos\theta}$
and $PB=\sqrt{r^2+\lambda^2-2r\lambda\cos\theta}$
$PA^2-PB^2=4r\lambda\cos\theta$
$PA-PB=\frac{4r\lambda\cos\theta}{(PA+PB)}=2\lambda\cos\theta$
[$\because$ PA = r, PB = r]
For P to have maximum intensity, $2\lambda\cos\theta=n\lambda$
$\cos\theta$ = n/2 where n is integer for n = 0, θ = 90°, 270°
n = ± 1, θ = 60°, 120°, 240°, 300°
n = ± 2, θ = 0°, 180°
So, positions of maxima are at θ = 0°, 60°, 90°, 120°, 180°, 240°, 270° and 300° i.e., 8 positions will be obtained.
24. **Ans.** 3 **Reason :** $mg+\frac{d}{dt}(M\times V)$
$$=mg+m\times\sqrt{2gy}\times\frac{d}{dt}=mg+\frac{m\times2g}{\sqrt{2gy}}\times\frac{dy}{dt}$$
but $\frac{dy}{dt}=\sqrt{2gy}$ = mg + 2mg = 3 mg.
25. **Ans.** 0 **Reason :** As x = $(t-3)^2$ ……….. (1)
So v = (dx/dt) = 2(t – 3) …………. (2)
(a) v will be zero when 2(t – 3) = 0, i.e., t = 3
Substituting this value of t in equation (1), x = $(3-3)^2$ = 0
i.e., when velocity is zero, displacement is also zero.

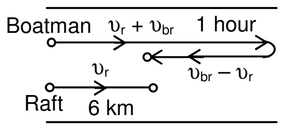


26. **Ans.** 6 **Reason :** As the rods are in series, $R_{eq}=R_A+R_B+R_C$ with R = (L/KA)
i.e., $R_{eq}=\frac{L}{2KA}+\frac{L}{KA}+\frac{L}{0.5KA}=\frac{7L}{2KA}$ ……… (1)
And hence, $H=\frac{dQ}{dt}=\frac{\Delta\theta}{R}=\frac{(100-\theta)}{(7L/2KA)}=\frac{200KA}{7L}$
Now in series, rate of flow of heat remains same, i.e., H = $H_A=H_B=H_C$.
So for rod A,
$\left[\frac{dQ}{dt}\right]_A=\left[\frac{dQ}{dt}\right]$, i.e. $\frac{(100-T_{AB})2KA}{L}=\frac{200KA}{7L}$
or T_{AB} = 100 – (100/7) = (600/7) = 87.7° C
And for rod C,
$\left[\frac{dQ}{dt}\right]_C=\left[\frac{dQ}{dt}\right]$, i.e. $\frac{(T_{BC}-0)\times0.5KA}{L}=\frac{200KA}{7L}$
or T_{BC} = (400/7) = 57.1°C
Furthermore if K_{eq} is equivalent thermal conductivity,
$R_{eq}=\frac{L+L+L}{K_{eq}A}=\frac{7L}{2KA}$ [From equation (1)]
i.e. K_{eq} = (6/7) K.
27. **Ans.** 3 **Reason:** Let υ_{br} be the velocity of boatman with respect to river or velocity of boatman in still water and υ_r the river velocity. Velocity of raft is also υ_r.
Assuming river to be at rest or raft to be at est. The boatman will move with υ_{br} both during its downstream and upstream motion. Therefore
Time of upstream motion = time of downstream motion = 1 hr.

$\therefore$ Total time of motion of raft = 2 hrs. In this 2 hrs, the raft moves 6 km. Hence the raft velocity or river velocity is 3 km/hr.



Assuming river to be at rest

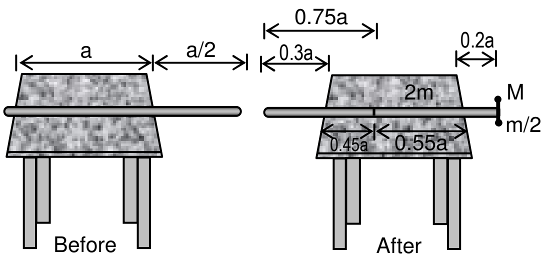


Actual motion of boatman and raft

28. **Ans.** 5 **Reason:** Let x be the displacement of straw when the first insect reaches the opposite end. Hence displacement of insect would be $\left(\frac{3}{2}a - x\right)$. For center of mass to remain stationary we have -

$2m(x) = \frac{m}{2}\left(\frac{3}{2}a - x\right)$ or $x = 0.3a$

Therefore, the situation is as follows



Let M be the mass of second insect.

For the straw not to topple the centre of mass (of straw + two insects) should lie inside the table or

$2m\ (0.55a) = \left(M + \frac{m}{2}\right)\ (0.2\ a)$

or m = 5 m.

CHEMISTRY

29. **Ans.** 0.30 g
30. **Ans.** 30%
31. **Ans.** Frenkel defect
32. **Ans.** BH_3 . THF / H_2O_2 . NaOH, Hg $(OAc)_2$ / $NaBH_4$. NaOH, H_2O / H^+
33. **Ans.** All except I.
34. **Ans.** Li, Mg or Al
35. **Ans.** $\frac{M}{8}$
36. **Ans.** orthophosphate
37. **Ans.** (A, B)
38. **Ans.** (A, B, C) **Reason :** The reaction

$2X + B_2H_6 \longrightarrow [BH_2(X)_2]^+ [BH_4]^-$

proceeds when X = NH_3, CH_3NH_2 or $(CH_3)_2NH$.

39. **Ans.** (A, B C, D) **Reason :** All being alkanes have only sp^3-hybridized carbons.
40. **Ans.** (B, C) **Reason :** Frenkel defect is also called dislocation defect because smaller ions are dislocated from their lattice sites into the interstitial sites. Hence (b) is correct. Trapping of an electron leads to the formation of F-centre. Hence, (c) is correct.
41. **Ans.** (A, B) **Reason:** $(Ph_3P)_3$ RhCl is Wilkinson's catalyst and $TiCl_4 + (C_2H_5)_3$ Al is Ziegler-Natta catalyst.
42. **Ans.** $Pb^{2+} \longrightarrow Pb^{4+}$ **Reason :** During recharging the coil, cell reaction goes in reverse direction.
43. **Ans.** 98 **Reason :** Two moles of H_2SO_4 are used in the net cell reaction in which two electrons are exchanged through he cell reaction.
44. **Ans.** $CH_3 - CH = CH - CN$
45. **Ans.** $CH_3 - CH_2 - CH_2 - CHO$
46. **Ans.** $CH_3 - CH = CH - COOH$
47. **Ans.** 5 **Reason :** $H_3PO_3 \longrightarrow 2H^+ + HPO_3^{2-}$; $\Delta_rH = ?$

$2H^+ + 2OH^- \longrightarrow 2H_2O$;

$\Delta_rH = -55.84 \times 2 = -111.68$

$-106.68 = \Delta_{ion} H - 55.84 \times 2$

$\Delta_{ion} H = 5$ kJ/mol

48. **Ans.** 2 **Reason :** ΔE =

$\frac{hc}{\lambda} = \frac{(6.63\times10^{-34} J-s)(3.00\times10^{8} m/s)}{3.055\times10^{-8} m}$

$= 6.52 \times 10^{-18}$ J

$\Delta E_H = \frac{3}{4}(2.176 \times 10^{-18}\ J)$

$= 1.63 \times 10^{-18}$ J; $\Delta E = \Delta E_H\ (Z^2)$

$Z^2 = \frac{\Delta E}{\Delta E_H} = \frac{(6.52\times10^{-18})}{(1.63\times10^{-18})} = 4$;

Z = 2 (helium)

49. **Ans.** 3 **Reason :** $\frac{A_{0(X)}}{A_{0(Y)}} = \frac{4}{1}$; $\frac{A_x}{Ay} = 1$

$\lambda_y - \lambda_x = \frac{1}{t}\ln\left(\frac{A_{0(X)}}{A_{0(Y)}} \times \frac{A_x}{Ay}\right)$

$(\lambda_y - \lambda_x)t = \ln\left(\frac{1}{4}\right)$; $(t_{1/2})_y = 30$ min.

50. **Ans.** 4 **Reason :** Octahedral void present at the centre of cube and tetrahedral void is present at $(1/4)^{th}$ of the distance along each body diagonal.

$\therefore \frac{\sqrt{3}a}{2} = 2 \times$ distance between octahedral and tetrahedral void.

51. **Ans.** 2
52. **Ans.** 3
53. **Ans.** 1
54. **Ans.** 6 **Reason :** $3Cl_2 + 6OH^- \longrightarrow 5Cl^- + ClO_3^- + 3H_2O$
55. **Ans.** 2
56. **Ans.** 1

MATHEMATICS

57. **Ans.** $-\overline{W}$ **Reason:** We have to find Z in terms of W under given condition.
$|Z| = |W| = r$ say
Let $W = re^{i\theta}$ $\therefore$ $\overline{W} = re^{-i\theta}$
and arg Z = π – arg $\overline{W}$ = $\pi - \theta$
$\therefore Z = re^{i(\pi-\theta)} = re^{i\pi}.e^{-i\theta} = -re^{-i\theta} = \overline{W}$.

58. **Ans.** 1 **Reason:** Solve the given equations for a and d
a = d = 1/mn
$\therefore T_{mn} = a + (mn - 1)d = 1/mn\,(1 + mn - 1) = 1$.

59. **Ans.** 0 **Reason:** Express as quadratic and show $\Delta < 0$.

60. **Ans.** $\binom{n+2}{r}$ **Reason:** ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$
$= ({}^nC_r + {}^nC_{r-1}) + ({}^nC_{r-1} + {}^nC_{r-2})$
$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$.

61. **Ans.** 1/5 **Reason:** AB = I if $a_{ij} = 0$, $i \neq j$ and $a_{ij} = 1$ for $i = j$.
$a_{11} = 5\lambda = 1 \Rightarrow \lambda = 1/5$, $a_{12} = 28\lambda - 28\lambda = 0$ $a_{13} = 14\lambda - 14\lambda = 0$
You may verify other elements also satisfy the criteria.

62. **Ans.** P (E) = 1/3, P (F) = 1/4 or P (E) = 1/4, P (F) = 1/3
Reason: Since E and F are independent, we have
$P(E \cap F) = P(E)\,P(F)$
$\therefore P(E)\,P(F) = 1/12$ … (1)
E and F are independent
$\therefore E^c$ and F^c are also independent
$\therefore P(E^c \cap F^c) = P(E^c).\,P(F^c) = 1/2$
$\Rightarrow (1 - P(E)).(1 - P(F)) = ½$
$\Rightarrow 1 - P(E) - P(F) + P(E)\,P(F) = 1/2$
$\Rightarrow 1 - P(E) - P(F) + 1/12 = 1/2$
$\Rightarrow P(E) + P(F) = 7/12$ … (2)
Solving (1) and (2), we get P (E) = 1/4, 1/3
Then P (F) = 1/3, 1/4.

63. **Ans.** $\tan\beta + 2\tan\gamma$ **Reason:** $\alpha + \beta = \pi/2 \Rightarrow \alpha = \pi/2 - \beta$
or $\tan\alpha = \cot\beta$
or $\tan\alpha\tan\beta = 1$ … (1)
$$\therefore\ \tan\gamma = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{\tan\alpha - \tan\beta}{2}$$
$\therefore \tan\alpha = \tan\beta + 2\tan\gamma$.

64. **Ans.** 60º **Reason:** Δ = 1/2 ab = 1/2 p. 4p i.e. 1/2 b.h
$\therefore ab = 4p^2$ Also $a^2 + b^2 = c^2 = 16p^2$
$\therefore (a - b)^2 = a^2 + b^2 - 2ab$
$= 16p^2 - 8p^2 = 8p^2$
Also $(a + b)^2 = a^2 + b^2 + 2ab = 24p^2$
$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2} = \frac{1}{\sqrt{3}}.1$$
$$\therefore\ \frac{A-B}{2} = 30^\circ \Rightarrow A - B = 60º.$$

65. **Ans.** • $f\left(\frac{\pi}{2}\right) = -1$ • $f(-\pi) = 0$ **Reason:** $\pi^2 = 9$.
Something $\Rightarrow [\pi^2] = 9$, $[-\pi^2] = -10$
$\therefore f(x) = \cos(9x) + \cos(-10x)$
$\Rightarrow f(x) = \cos(9x) + \cos(10x)$ … (1)
$$(1) \Rightarrow\ f\left(\frac{\pi}{2}\right) = \cos\left(\frac{9\pi}{2}\right) + \cos\left(\frac{10\pi}{2}\right)$$
$$= \ = \cos\left(4\pi + \frac{\pi}{2}\right) + \cos(5\pi)$$
$$= \ = \cos\frac{\pi}{2} - 1 = -1 \text{ as } \cos(n\pi) = (-1)^n$$
$f(\pi) = \cos(9\pi) + \cos(10\pi) = -1 + 1 = 0$
$f(-\pi) = \cos(-9\pi) + \cos(-10\pi) = -1 + 1 = 0$
$f(\pi/4) = \cos(9\pi/4) + \cos(10\pi/4) = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$.

66. **Ans.** • 2 • 1 **Reason:** $A = \lim_{m,\,n\to\infty} 1 + \cos^{2m}(\pi x.n!)$
I. If $|\cos\theta| < 1$, then A = 1 + 0 = 1
II. If $|\cos\theta| = 1$, then A = 1 + 1 = 2.

67. **Ans.** • $\frac{dy}{dx} = \frac{1}{a + b\cos x}$ • $\frac{d^2y}{dx^2} = \frac{b\sin x}{(a + b\cos x)^2}$
Reason: $y = \frac{2}{\sqrt{a^2 - b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$
$$\Rightarrow\ \frac{dy}{dx} = \frac{2}{\sqrt{(a^2-b^2)}}.\frac{1}{1+\left(\frac{a-b}{a+b}\right).\tan^2\left(\frac{x}{2}\right)}.\sec^2\left(\frac{x}{2}\right).\left(\frac{a-b}{a+b}\right)^{1/2}.\frac{1}{2}$$
$$\Rightarrow\ \frac{dy}{dx} = \frac{1}{(a+b)}.\frac{(a+b)}{(a+b)\cos^2\left(\frac{x}{2}\right) + (a-b)\sin^2\left(\frac{x}{2}\right)}$$
$$= \frac{1}{a\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + b\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)}$$
$$\Rightarrow\ \frac{dy}{dx} = \frac{1}{a + b\cos x} \ \Rightarrow\ \frac{d^2y}{dx^2} = \frac{b\sin x}{(a + b\cos x)^2}.$$

68. **Ans.** • $\frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \frac{1}{2^3}\tan\frac{x}{2^3}....\text{to } \infty = -\cot x + \frac{1}{x}$
• $\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + ... + \ \infty = \operatorname{cosec}^2 x - \frac{1}{x^2}$
Reason: Taking log on both sides of the given equation,
$\log\ \cos\left(\frac{x}{2}\right) + \log\ \cos\left(\frac{x}{2^2}\right) + \log\ \cos\left(\frac{x}{2^3}\right) + ... = \log$
$\sin x - \log x$
Differentiating w.r.t.x,
$-\frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \frac{1}{2^3}\tan\frac{x}{2^3}....\text{to } \infty = \cot x - \frac{1}{x}$ or
$\frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \frac{1}{2^3}\tan\frac{x}{2^3}....\text{to } \infty = -\cot x + \frac{1}{x}$

Again differentiating w.r.t.x,

$$\frac{1}{2^2}\sec^2\frac{x}{2}+\frac{1}{2^4}\sec^2\frac{x}{2^2}+....+\ \infty=\operatorname{cosec}^2x-\frac{1}{x^2}.$$

69. **Ans.** • velocity is max. at $t=\frac{6-2\sqrt{3}}{3}$ ● acceleration is min. at t = 2 ● min. distance is at t = 0, 4 **Reason:** x =

$\frac{t^4}{4}-2t^3+4t^2+7$ … (1)

$\frac{dx}{dt}=v=t^3-6t^2+8t$ … (2)

$f=\frac{d^2x}{dt^2}=\text{Acc.}=3t^2-12t+8$ … (3)

I. v is max. $\frac{dv}{dt}=0$, $\frac{d^2v}{dt^2}<0$ or if f =0 and 6t –2 < 0… (4)

$f = 0 \Rightarrow 3t^2 - 12t + 8 = 0$

$\Rightarrow\ t=\frac{12\pm(144-96)^{1/2}}{6}=\frac{12\pm4\sqrt{3}}{6}=\frac{6\pm2\sqrt{3}}{3}$

$\therefore$ If $t=\frac{6+2\sqrt{3}}{3}$, then $6t-12=4\sqrt{3}>0$, (not possible)

$\therefore$ If $t=\frac{6-2\sqrt{3}}{3}$, then $6t-12=-4\sqrt{3}<0$, which is true.

II. Acc. Is minimum if $\frac{df}{dt}=0$ and $\frac{d^2f}{dt^2}>0$

$\Rightarrow 6t-12=0$, $\frac{d^2f}{dt^2}=6>0$, by (3) $\Rightarrow t=2 \Rightarrow$ (b) is true.

III. x is min if $\frac{dx}{dt}=0$ and $\frac{d^2x}{dt^2}>0$

or if $t^3-6t^2+8t=0$ … (5)

and $3t^2-12t+8>0$ … (6)

From (4) $\Rightarrow t(t^2-6t+8)=0$

$\Rightarrow (t-4)(t-2)t=0 \Rightarrow t = 0, 2, 4.$

Out of these only t = 0, t = 4 satisfies

$\Rightarrow$ (c) is true.

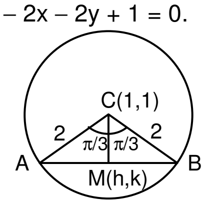
70. **Ans.** $x^2+y^2-2x-2y+1=0$ **Reason:** The centre of given circle is (1, 1) and radius is 1.
The chord AB whose mid-point is M (h, k) subtends an angle of 120º at C.

$\therefore\ \frac{CM}{AC}=\cos60^\circ=\frac{1}{2}$

$\therefore\ 4CM^2 = AC^2$

or $4\{(h-1)^2+(k-1)^2\}=4$

$\therefore$ Locus is $x^2+y^2-2x-2y+1=0$.



71. **Ans.** (1/2, 1/4) **Reason:** Let (h, k) be any point on the given line

$\therefore$ 2h + k = 4 and c.c. is hx + ky = 1

or hx + (4 – 2h) y = 1 or (4y –1) + h (x – 2y) =0

P + λQ = 0. It passes through the intersection of P = 0 and Q = 0 of (1/2, 1/4) .

72. **Ans.** $\frac{5}{1+25x^2}$ **Reason:** y =

$\tan^{-1}\frac{5x-x}{1+5x.x}+\tan^{-1}\frac{\frac{2}{3}+x}{1-\frac{2}{3}x}$

$y=\tan^{-1}5x-\tan^{-1}x+\tan^{-1}2/3+\tan^{-1}x$

or $y=\tan^{-1}5x+\tan^{-1}2/3$ etc.

73. **Ans.** $\sqrt{\left(\frac{1-y^2}{1-x^2}\right)}$ **Reason:** Put x = sin θ and y = sin ϕ in the given equation.

$\therefore\ \frac{\cos\theta+\cos\phi}{\sin\theta-\sin\phi}=a$ or $\cot\frac{\theta-\phi}{2}=a$

$\therefore\ \theta-\phi=2\cot^{-1}a$

or $\sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$ = constant

Differentiate w.r.t.x, $\frac{1}{\sqrt{(1-x^2)}}-\frac{1}{\sqrt{(1-y^2)}}\frac{dy}{dx}=0$

$\therefore\ \frac{dy}{dx}=\sqrt{\left(\frac{1-y^2}{1-x^2}\right)}$.

74. **Ans.** $\frac{\sin^2(a+y)}{\sin a}$ **Reason:** sin y = x sin (a+ y),

$\therefore\ x=\frac{\sin y}{\sin(a+y)}$

Differentiate w.r.t. x.

$\therefore\ 1=\frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{\sin^2(a+y)}.\frac{dy}{dx}$

$\therefore\ \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin(a+y-y)}=\frac{\sin^2(a+y)}{\sin a}$.

75. **Ans.** 2 **Reason:** Here $\frac{z_1-2z_2}{2-z_1\bar{z}_2}=1$

$\Rightarrow\ |z_1-2z_2|=|2-z_1\bar{z}_2|$

$\Rightarrow\ |z_1-2z_2|^2=|2-z_1\bar{z}_2|^2$

$\Rightarrow (z_1-2z_2)(\overline{z_1-2z_2})=(2-z_1\bar{z}_2)(\overline{2-z_1\bar{z}_2})$

$\Rightarrow (z_1-2z_2)(\bar{z}_1-2\bar{z}_2)=(2-z_1\bar{z}_2)(2-\bar{z}_1z_2)$

$\Rightarrow\ z_1\bar{z}_1-2z_1\bar{z}_2-2\bar{z}_1z_2+4z_2\bar{z}_2$

$\Rightarrow\ |z_1|^2+4|z_2|^2=4+|z_1|^2.|z_2|^2$

$\Rightarrow\ (|z_1|^2-4)(1-|z_2|^2)=0$ But $|z_2|\neq1$ (given)

$\therefore\ |z_1|^2-4=0$ or $|z_1|=2$.

76. **Ans.** 3 **Reason:** Rewrite the given equation

$\sqrt{x+2\sqrt{x+2\sqrt{x+....+2\sqrt{x+2\sqrt{x+2x}}}}}=x$ … (1)

On replacing the last letter x on the L.H.S of Eq. (i) by the value of x expressed by Eq. (i) we obtain

$$x = \underbrace{\sqrt{x+2\sqrt{x+2\sqrt{x+....+2\sqrt{x+2x}}}}}_{\text{(2n radical signs)}}$$

Further, let us replace the last letter x by the same expression; again and again yields

$\therefore$ x =

$$x = \underbrace{\sqrt{x+2\sqrt{x+2\sqrt{x+....+2\sqrt{(x+2x)}}}}}_{\text{(4n radical signs)}}$$

we can write $x = \sqrt{x+2\sqrt{x+2\sqrt{x+....}}}$

$$= \lim_{N\to\infty} \sqrt{x+2\sqrt{x+2\sqrt{x+....+2\sqrt{x+2x}}}}$$

{N radial signs}

It follows that, $x = \sqrt{x+2\sqrt{x+2\sqrt{x+....}}}$

$$= \left|\lim_{x\to1-} f(x) - \lim_{x\to1+} f(x)\right|$$

Hence $x^2 = x + 2x \Rightarrow x^2 - 3x = 0$

Therefore x = 0, 3

$\therefore$ sum of roots = 0 + 3 = 3.

77. **Ans.** 4 **Reason:** Let $P = \dfrac{a+b}{2a-b} + \dfrac{c+b}{2c-b}$

$$= \frac{a+\dfrac{2ac}{a+c}}{2a-\dfrac{2ac}{a+c}} + \frac{c+\dfrac{2ac}{a+c}}{2c-\dfrac{2ac}{a+c}} \qquad \left(\because b = \frac{2ac}{a+c}\right)$$

$$= \frac{a+3c}{2a} + \frac{3a+c}{2c} = 1 + \frac{3}{2}\left(\frac{c}{a}+\frac{a}{c}\right) > 4$$

$$\left(\begin{array}{l}\because \dfrac{a}{c}+\dfrac{c}{a} > 2 \\ \therefore \dfrac{3}{2}\left(\dfrac{a}{c}+\dfrac{c}{a}\right) > 3\end{array}\right)$$

Also, $\sqrt{\lambda\sqrt{\lambda\sqrt{\lambda ...\infty}}} = \lambda^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...\infty} = \lambda^{\frac{1/2}{1-1/2}} = \lambda \quad \therefore \lambda = 4.$

78. **Ans.** 7 **Reason:** Number of ways of selecting n coupons consisting of C or A = 2^n ($\because$ the word CAT can not be written if at least one letters is not selected)

Now number of ways of selecting n coupons bearing only A = 1^n.

$\therefore$ Total number of ways = $2^n + 2^n + 2^n - 1^n - 1^n$

$= 3(2^n - 1)$

Given $3(2^n - 1) = 189$

$\Rightarrow 2^n - 1 = 63 \Rightarrow 2^n = 64 = 2^6$

$\therefore$ n = 6

then $\sum n^2 = \dfrac{n(n+1)(2n+1)}{6} = \dfrac{6.7.13}{6} = 91.$

79. **Ans.** 2 **Reason:** Applying $C_1 \to C_1 + C_2 + C_3$, then

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix} \quad (\because a^2 + b^2 + c^2 + 2 = 0)$$

Now, applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)^2$$

Hence, degree of f(x) = 2.

80. **Ans.** 0 **Reason:** $\because\ a^{\log_b c} = c^{\log_b a}$

$\therefore 3^{\log_5 7} - 7^{\log_5 3} = 0$

81. **Ans.** 1.

82. **Ans.** 3 **Reason:** Given If $f(x) + f\left(\dfrac{1}{1-x}\right) = \dfrac{2(1-2x)}{x(1-x)}$

$$= \frac{2}{x} - \frac{2}{1-x} \qquad \text{… (1)}$$

Replacing x by $\dfrac{1}{(1-x)}$, we obtain

$$f\left(\frac{1}{1-x}\right) + f\left(\frac{1}{1-\dfrac{1}{1-x}}\right) = 2(1-x) - \frac{2}{1-\dfrac{1}{1-x}}$$

$$\Rightarrow f\left(\frac{1}{1-x}\right) + f\left(1-\frac{1}{x}\right) = -2x + \frac{2}{x} \qquad \text{… (2)}$$

Again replacing x by $1-\dfrac{1}{x}$ in Eq. (1) we obtain

$$f\left(\frac{1}{1-x}\right) + f\left(\frac{1}{1-\left(1-\dfrac{1}{x}\right)}\right) = \frac{2}{\left(1-\dfrac{1}{x}\right)} - \frac{2}{1-\left(1-\dfrac{1}{x}\right)}$$

$$\Rightarrow f(x) - f\left(1-\frac{1}{x}\right) = 2x - \frac{2}{1-x} \qquad \text{… (4)}$$

Now adding (3) and (4) , then we get

$$2f(x) = \frac{2x}{x-1} - \frac{2}{1-x}$$

$$\therefore f(x) = \frac{x+1}{x-1}$$

$\therefore$ f(2) = 3.

83. **Ans.** 1 **Reason:** $\lim_{x\to0} e^{-2} \left\{\tan\left(\dfrac{\pi}{4}+x\right)\right\}^{1/x}$

$$= e^{-2}.e^{\lim_{x\to0}\frac{2\tan x}{(1-\tan x)}} \qquad (\text{form } 1^\infty)$$

$= e^{-2}.e^2 = 1.$

84. **Ans.** 9 **Reason:** Number of jumps = | LHL – RHL|

$$= \left|\lim_{x\to1-} f(x) - \lim_{x\to1+} f(x)\right|$$

$$= \left|\lim_{h\to0} f(1-h) - \lim_{h\to0} f(1+h)\right|$$

$$= \left|\left(\lim_{h\to0} 5 + (1-h)^2\right) - \left(\lim_{h\to0} f(1+h) - 4\right)\right|$$

= | (6 – (–3)| = 9.